



Congruent shapes

Two shapes are described as congruent if they are **identical**.

- The lengths of sides (edges) and sizes of angles must be equal between the two shapes for them to be congruent.
- Reflections or rotations change the orientation of a shape but they are still congruent to the original shape.
- A great way to check if two shapes are congruent is to place one on top of the other. If they match, with **no overlaps**, they are congruent.

Similar shapes

Two shapes are described as similar if one is an enlargement of the other. The shapes do not need to be orientated the same way.

- The sizes of angles must be equal between the two shapes.
- The shapes must also be proportionally the same. If one side on the enlarged shape doubles in length, all sides must be double the original size shape. The increase in size from one shape to another is called a scale factor.
- The shapes may need to be rotated to find out if they are similar. It is possible to calculate missing lengths on similar shapes when given either the scale factor or enough information to calculate it.

Perpendicular bisector construction of a line segment



Begin by drawing a straight horizontal line.

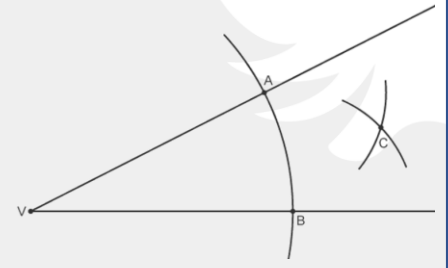
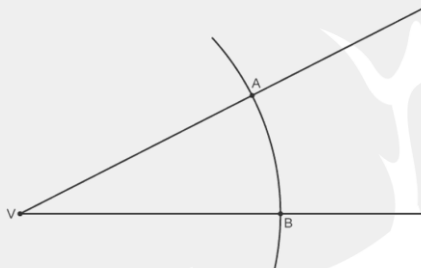
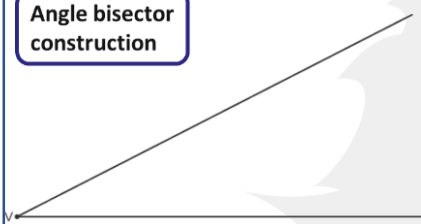


Placing your compass at one end of the line, draw an arc - making sure it is over half way across the line. Without changing the distance between the pencil and compass point, place your compass at the other end of the line and draw another arc - crossing the first arc in two places.

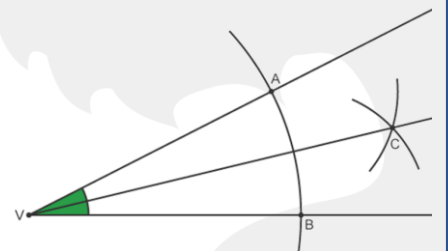


Use your ruler to draw a line connecting the points A and B. Point M is exactly halfway between X and Y. When drawn correctly, these lines will always be at right angles.

Angle bisector construction



1. Begin by drawing two lines, meeting at a point. Mark this point V.
2. Place your compass on the point where the lines meet, draw an arc that crosses both lines. Label the points A and B.
3. Place your compass on the point A, draw an arc towards the middle. Without changing the distance between your compass point and your pencil, place the compass on point B and draw an arc towards the middle - making sure that the two arcs cross. Mark this point C.
4. Finally, using a ruler connect point C to point V. Line VC bisects the angle AVC.

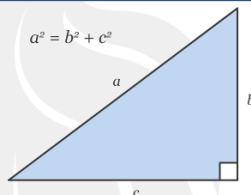




Pythagoras' Theorem

In a right-angled triangle there is a relationship among the three sides. This relationship is shown.

$$a^2 = b^2 + c^2$$



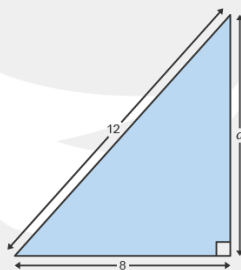
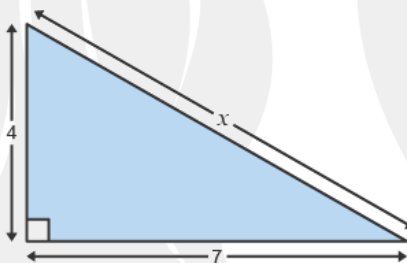
Write the equation $x^2 = 7^2 + 4^2$

Square the lengths you know $x^2 = 49 + 16$

Add together $x^2 = 65$

Find the square root $x = \sqrt{65}$

$x = 8.06$ (to 2 d.p.)



- Write the equation: $12^2 = a^2 + 8^2$
- Organise the equation $a^2 = 12^2 - 8^2$. To find the length of a short side, we can also use the formula $b^2 = a^2 - c^2$
- Square the lengths you know: $a^2 = 144 - 64$
- Do the subtraction: $a^2 = 80$
- Find the square root: $a = \sqrt{80}$
- $a = 8.94$ (to 2 d.p.)

Transformations

Key Concepts

A **reflection** creates a mirror image of a shape on a coordinate graph. The mirror line is given by an equation eg. $y = 2$, $x = 2$, $y = x$. The shape does not change in size.

A **rotation** turns a shape on a coordinate grid from a given point. The shape does not change size but does change orientation.

A **translation** moves a shape on a coordinate grid. Vectors are used to instruct the movement:

$\begin{pmatrix} x \\ y \end{pmatrix}$ → Positive - Right
 → Negative - Left
 → Positive - Up
 → Negative - Down

An **enlargement** changes the size of an image using a scale factor from a given point.

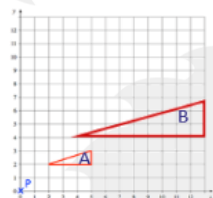
A **positive scale factor** will increase the size of an image.

A **fractional scale factor** will reduce the size of an image.

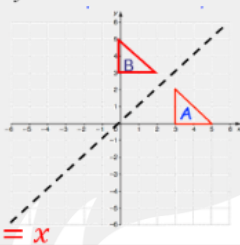
A **negative scale factor** will place the image on the opposite side of the centre of enlargement, with the image inverted.

Examples

Enlarge shape A by scale factor 2 from point P.



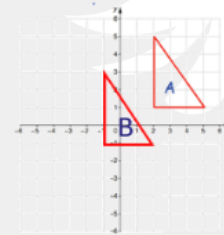
Reflect shape A in the line $y = x$. Label it B.



Rotate shape B from the point $(-1, -2)$



Translate shape A by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$. Label it B.



Describe the **single** transformation you see on each coordinate grid from A to B:



ANSWERS: a) reflection, $y = 1$ b) reflection $y = x$ c) rotation, centre $(0,0)$, 90° anticlockwise d) translation $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ e) enlargement, centre $\frac{1}{2}$ f) scale factor 2 g) enlargement, centre $(1,-2)$ scale factor $\frac{1}{2}$

Key Words

	Rotate	Clockwise	Anticlockwise			
Centre	Degrees	Reflect	Translate	Vector	Scale Factor	



Simplifying SURDS

Simplify $\sqrt{12}$

$$= \sqrt{4 \times 3}$$

$$= \sqrt{4} \times \sqrt{3}$$

$$= 2\sqrt{3}$$

Simplify $\sqrt{48}$

$$= \sqrt{16} \times \sqrt{3}$$

$$= 4\sqrt{3}$$

Rationalising the denominator

Rationalising an expression means getting rid of any surds from the bottom (denominator) of fractions.

Usually when you are asked to simplify an expression it means you should also rationalise it.

Example

Simplify $\frac{4}{\sqrt{3}}$

Answer

To rationalise the denominator, multiply the fraction by $\frac{\sqrt{3}}{\sqrt{3}}$

$$= \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{4\sqrt{3}}{3} \text{ (Remember } \sqrt{3} \times \sqrt{3} = 3\text{)}$$

Sometimes the denominator might be more complicated and include other numbers as well as the surd.

If this is the case you need to multiply the fraction by a number that will cancel out the surd. Remember to multiply the numerator by the same number or you will change the value of the fraction.

Example

Express $\frac{12}{4+\sqrt{7}}$ with a rational denominator

Answer

To rationalise the denominator, multiply the fraction by $\frac{4-\sqrt{7}}{4-\sqrt{7}}$

$$= \frac{12}{4+\sqrt{7}} \times \frac{4-\sqrt{7}}{4-\sqrt{7}}$$

$$= \frac{12(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}$$

Simplify the denominator by multiplying out the brackets to give $16 - 4\sqrt{7} + 4\sqrt{7} - 7$. The $-4\sqrt{7}$ and $+4\sqrt{7}$ cancel out. This leaves the denominator as 9.

$$= \frac{12(4-\sqrt{7})}{9}$$