

SUBJECT: Maths

UNIT:

Year 10 Number & Data

SANDHILL VIEW

EST. 1993



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Key Concepts

All students should be able to utilise the principles of basic probability, applying it to probability of singular and multiple events. They will learn how to present data in statistical diagrams enabling further probabilities to be calculated

Each day Paul wears either a black tie or a red tie to work.

On any day the probability he wears a black tie is $\frac{5}{9}$

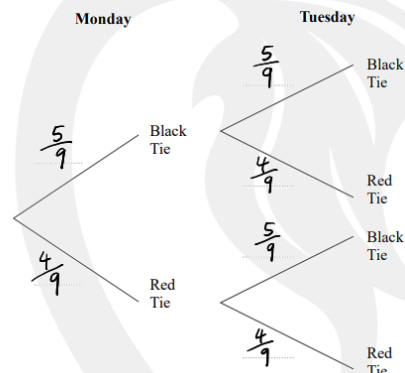
(a) Complete the probability tree diagram for Monday and Tuesday.

Work out the probability Paul wears different coloured ties on Monday and Tuesday.

Black, Red or Red, Black

$$\frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9}$$

$$\frac{20}{81} + \frac{20}{81} = \frac{40}{81}$$



Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B :

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)P(B)$$

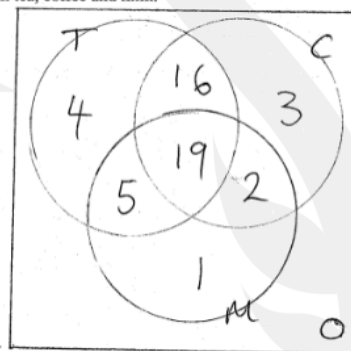
Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks
19 people like all three drinks.
16 people like tea and coffee but do not like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.

Sami selects at random one of the 50 people.

Work out the probability that this person likes tea.

$$\frac{44}{50}$$



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Inequalities

Inequalities are the relationships between two expressions which are not equal to one another. The symbols used for inequalities are $<$, $>$, $\leq$, $\geq$.

$7 > x$ reads as '7 is greater than x ' (or ' x is less than 7', reading from right to left).

$x \leq -4$ reads as ' x is less than or equal to -4' (or '-4 is greater than or equal to x ', reading from right to left).

Inequalities on a number line

Inequalities can be shown on a **number line**.

Open circles are used for numbers that are **less than or greater than** ($<$ or $>$).

Closed circles are used for numbers that are **less than or equal to and greater than or equal to** ($\leq$ or $\geq$).

For example, this is the number line for the inequality $x \geq 0$:



The symbol used is greater than or equal to ($\geq$) so a closed circle must be used at 0. x is greater than or equal to 0, so the arrow from the circle must show the numbers that are **larger** than 0.

Example

Show the inequality $y < 2$ on a number line.

y is less than ($<$) 2, which means an open circle at 2 must be used. y is less than 2, so an arrow below the values of 2 must be drawn in.



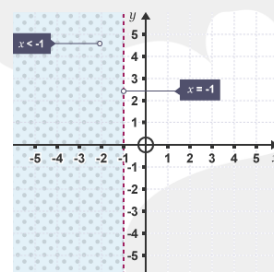
Graphs of inequalities - Higher

An inequality can be represented graphically as a region on one side of a line.

Key fact

Inequalities that use $<$ or $>$ symbols are plotted with a dashed line to show that the line is not included in the region. Inequalities that use $\leq$ or $\geq$ symbols are plotted with a solid line to show that the line is included in the region.

For example, this graph shows the inequality $x < -1$. This can be seen as there is a dashed line at $x = -1$, and the region where the x coordinates are less than -1 is shaded.



Example

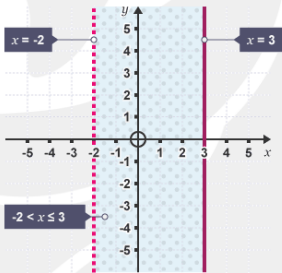
Show the region satisfied by the inequality $-2 < x \leq 3$.

Identify the two regions shown by the inequalities. These are $-2 < x$ (or $x > -2$) and $x \leq 3$.

$x > -2$: draw a dotted line at $x = -2$. $x = -2$ is the graph made by coordinates points where x is equal to -2, for example $(-2, 5)$, $(-2, 4)$, $(-2, 3)$, $(-2, 2)$ and so on.

$x \leq 3$: draw a solid line at $x = 3$. $x = 3$ is the graph made by coordinate points where x is equal to 3, for example $(3, -4)$, $(3, -3)$, $(3, -2)$, $(3, -1)$ and so on.

x is the values in between these two inequalities, so shade this region.



Solving inequalities

The process to **solve inequalities** is the same as the process to **solve equations**, which uses inverse operations ❶ to keep the equation or inequality balanced. Instead of using an equals sign, however, the inequality symbol is used throughout.

Example

Solve the inequality $3m + 2 > -4$.

The inequality will be solved when m is isolated on one side of the inequality. This can be done by using inverse operations at each stage of the process.

$$\begin{array}{rcl} 3m + 2 & > & -4 \\ -2 & & -2 \\ 3m & > & -6 \\ \div 3 & & \div 3 \\ m & > & -2 \end{array}$$



The final answer is $m > -2$, which means m can be any value that is bigger than -2, not including -2 itself. If this answer was to be placed on a number line, an open circle would be needed at -2 with a line indicating the numbers that are greater than 2.

Integer solutions to inequalities

When solving inequalities there will be a range of answers. Because any numbers represented by the range are acceptable, there are an infinite ❷ amount of solutions to inequalities.

For example, if $a > 3$, then any number that is bigger than 3 is a possible answer, from any decimal slightly bigger than 3 to infinity.

Sometimes, only integer ❸ solutions are required.

Example

List the integer solutions to $-5 < k < 1$.

This can be read as -5 is less than k , which is less than 1.

That means that k is larger than -5, but not equal to -5, so the smallest integer that k can be is -4.

k is less than 1, but not equal to 1, so the largest integer that k can be is 0.

k can also be the integers between -4 and 0.

This means that the integer solutions to $-5 < k < 1$ are: -4, -3, -2, -1, 0.

Finding the mean from a table

The **mean** is found by adding up all the numbers and dividing by how many numbers there are.

To find the mean in this example, the total number of goals must be found and then divided by the number of games.

From the table, it can be seen that in 2 games no goals were scored. This makes a grand total of zero goals so far. The rest of the total amount of goals can be worked out in this way, by multiplying goals (x) by the frequency (f).

Call this column fx (f multiplied by x).

	Number of goals (x)	Frequency (f)	fx
	0	2	$0 \times 2 = 0$
	1	2	$1 \times 2 = 2$
	2	5	$2 \times 5 = 10$
	3	1	$3 \times 1 = 3$
Total		10	15

The total number of goals is 15. There were 10 football games so $15 \div 10 = 1.5$.

The mean number of goals is 1.5 goals per game.

Remember to divide fx by the total of the frequencies, not by the amount of different items of data – the correct answer here is $\frac{15}{10}$ not $\frac{15}{4}$.

Estimate the **mean number** of minutes late.

In grouped tables the exact number of minutes late cannot be found. As the data has been grouped, the exact data values are not known so only an estimate of the mean can be found.

To find the mean number in this frequency table, divide the total number of minutes late by the total number of trains.

To estimate the number of minutes late for each group, create a midpoint column. To find midpoints, add the start and end points and then divide by 2.

The midpoint of 0 and 4 is 2, because $\frac{0+4}{2} = \frac{4}{2} = 2$.

Now an estimate for the number of minutes late is known, the total number of minutes late can be found by multiplying the frequencies by the midpoints.

Number of minutes late (m)	Frequency (f)	Midpoint (x)	Total minutes late (fx)
$0 < m \leq 4$	11	2	$11 \times 2 = 22$
$4 < m \leq 8$	13	6	$13 \times 6 = 78$
$8 < m \leq 12$	7	10	$7 \times 10 = 70$
$12 < m \leq 16$	9	14	$9 \times 14 = 126$
$16 < m \leq 20$	4	18	$4 \times 18 = 72$
Total = 44			Total = 368

Now find the estimate for the mean by dividing the total minutes late by the total number of trains.

$$\text{mean} \approx \frac{368}{44} = 8.36 \text{ (2 dp)}$$